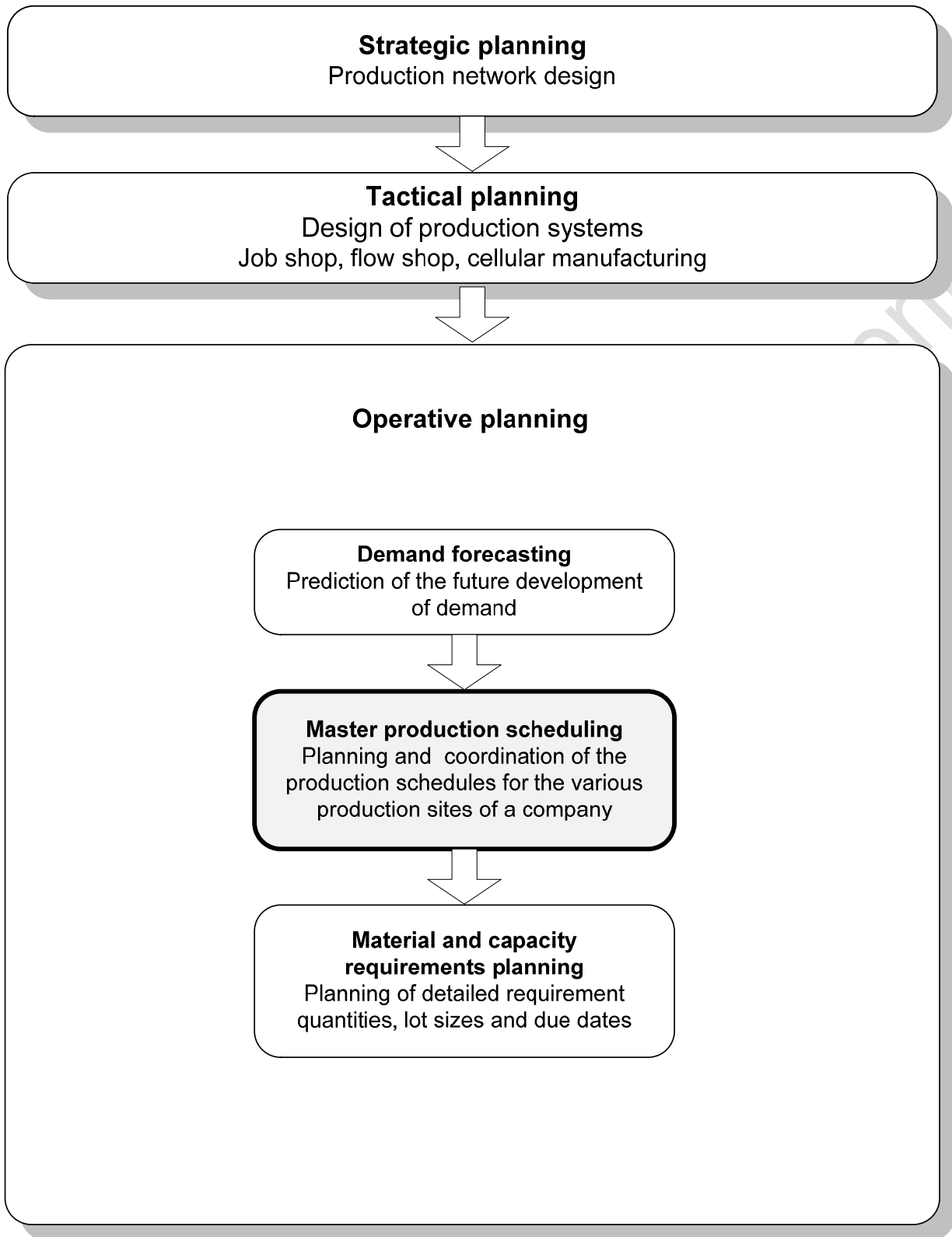


# 5. Master production scheduling

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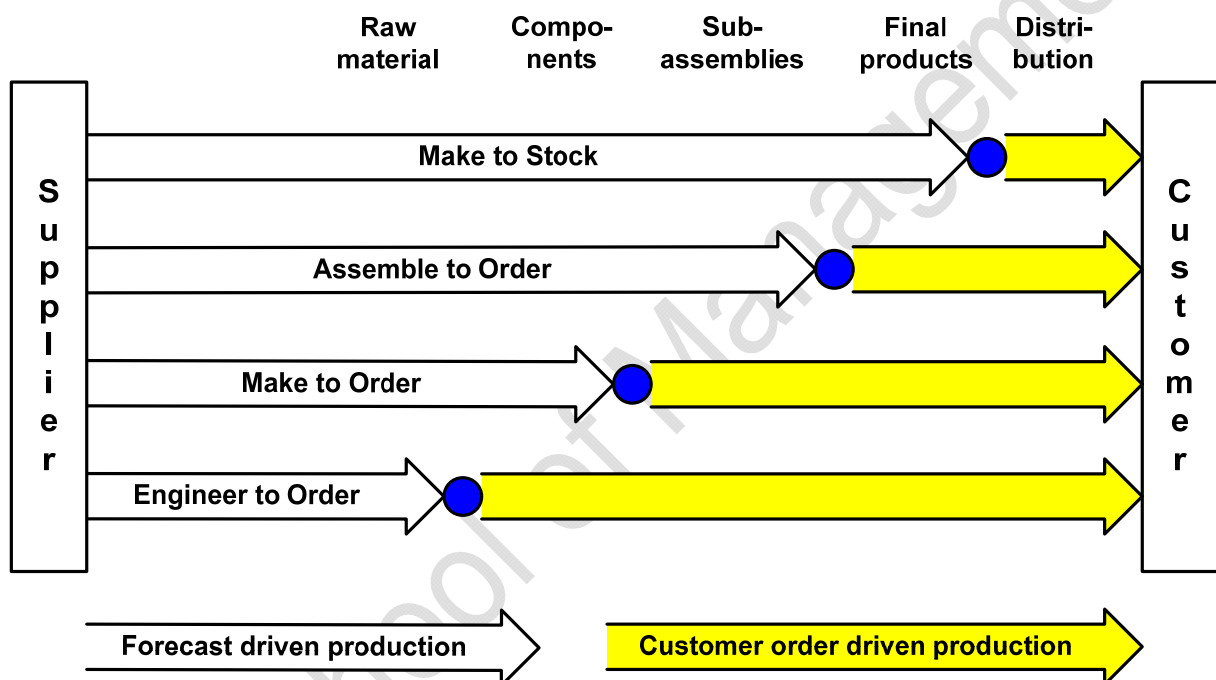


## 5. Master production scheduling

- **Task:**

Development of plant-specific production schedules considering potential capacity bottlenecks.  
Basis for material requirements planning.

- Planning horizon: **3-12 months**
- Result: **Specific production quantities for final products and key sub-assemblies**
- **Types of production systems categorized by the customer order decoupling point**



❓ Characterize each of the above production systems and name typical final products for each of the systems.

⇒ Non-order driven production

(1) „Make-to-stock“:

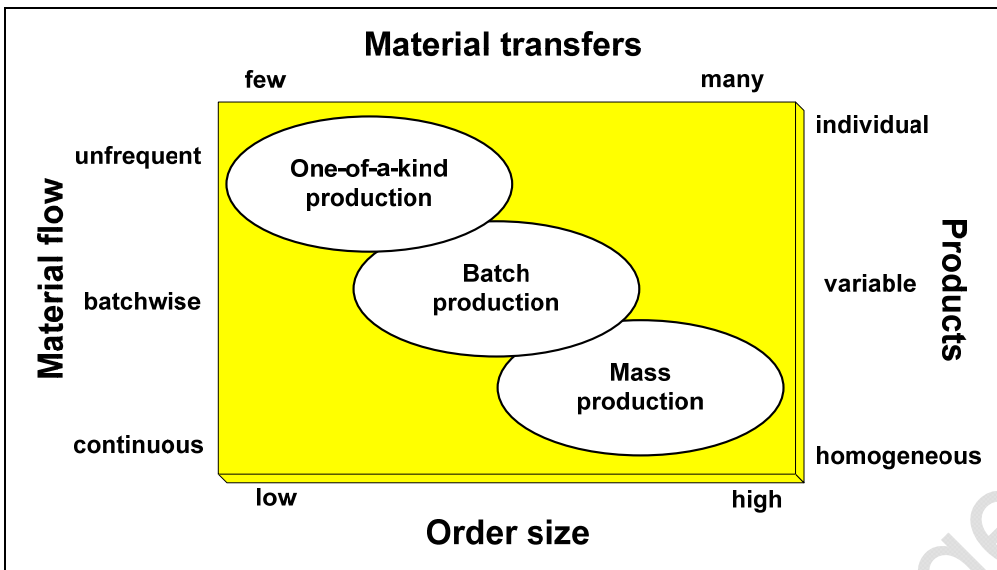
⇒ Order driven production

(2) „Assemble to order“:

(3) „Make to order“:

(4) „Engineer to order“:

• **Process types of production**



❖ **Characterize the process types above according to the variety of the manufactured products and the types of production processes.**

• **One-of-a-kind production**

a) Product variety:

b) Production processes:

• **Batch production**

a) Product variety:

b) Production processes:

• **Mass production**

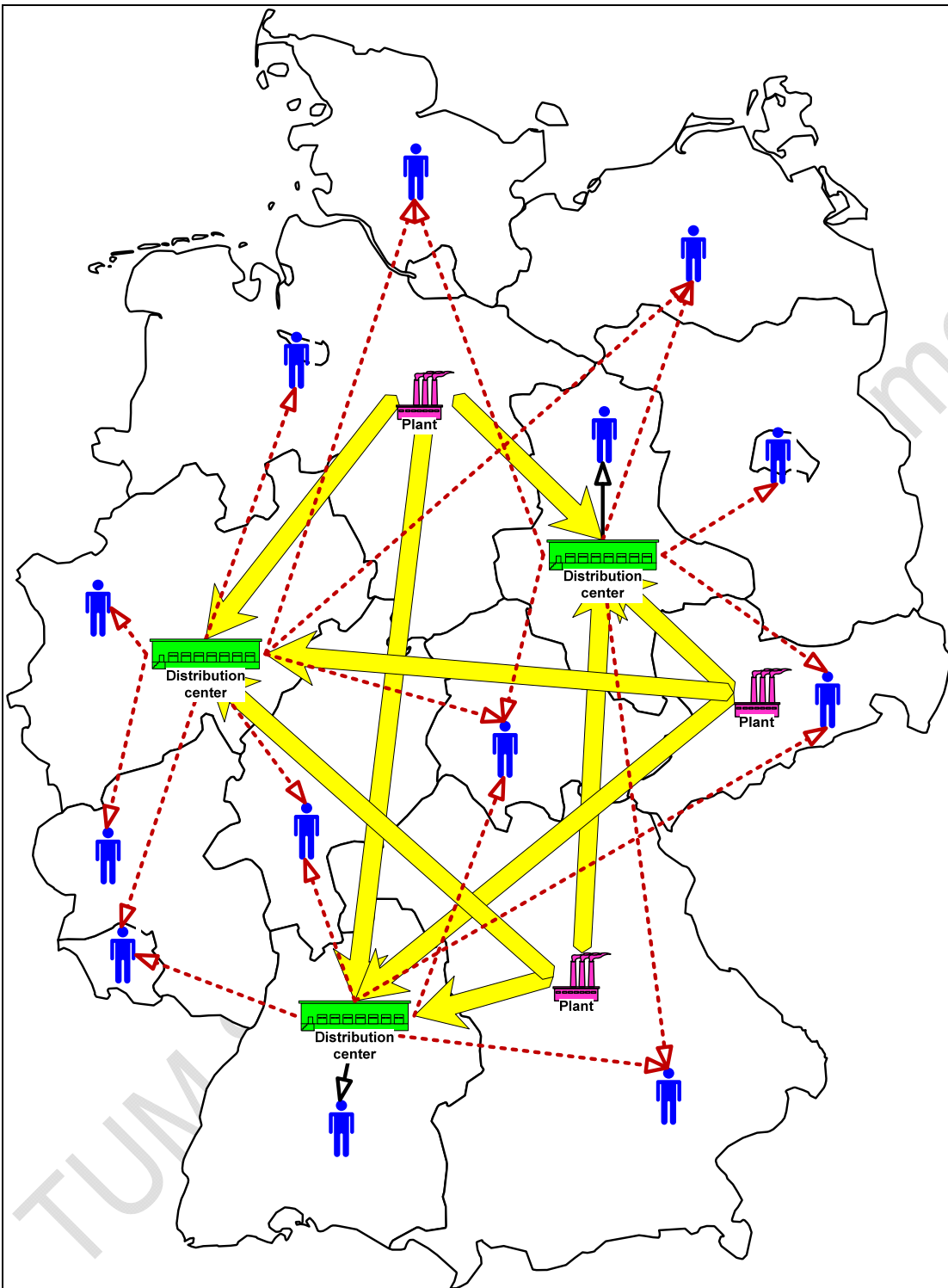
a) Product variety:

b) Production processes:

❖ **Assign the types of products to one of the process types.**

Product	One-of-a-kind production	Batch production	Mass production
Machines			
Hair dyes			
Cars			

## 5.1 Mass production



- **Characteristics of mass production**

- ⇒ Typically several production sites and distribution centers
- ⇒ Integration of suppliers, production and distribution (**Supply Chain Management**)
- ⇒ Make-to-stock or assemble-to-order systems

**◇ ? Bottleneck analysis**

A production system for the manufacture of two products consists of five stages. Product 1 visits stages A, B, C and E while product 2 visits stages A, D and E. Production capacities for stages A, B, C, D, E are 8, 5, 6, 7 and 10 time units, respectively. Capacity requirements per product unit are 0.5 time units in all stages for product 1 and 1.0 time units for product 2. Per unit profit is 8€ and 12€ for product 1 and 2, respectively. Sales limit is 8 units for each product type.

a) Display the material flow and the respective capacities graphically.

b) Formulate the decision problem as a linear optimization model. Are some of the constraints redundant?

⇒ **Decision variables**

$x_1, x_2$  = production quantity of product 1 and 2, respectively

⇒ **Model formulation**

Maximize

Subject to

Capacity

Sales

Non-negativity

⇒ **Redundant constraints:**

⇒ **Reduced model formulation**

Maximize

Subject to

c) Determine the optimal solution.

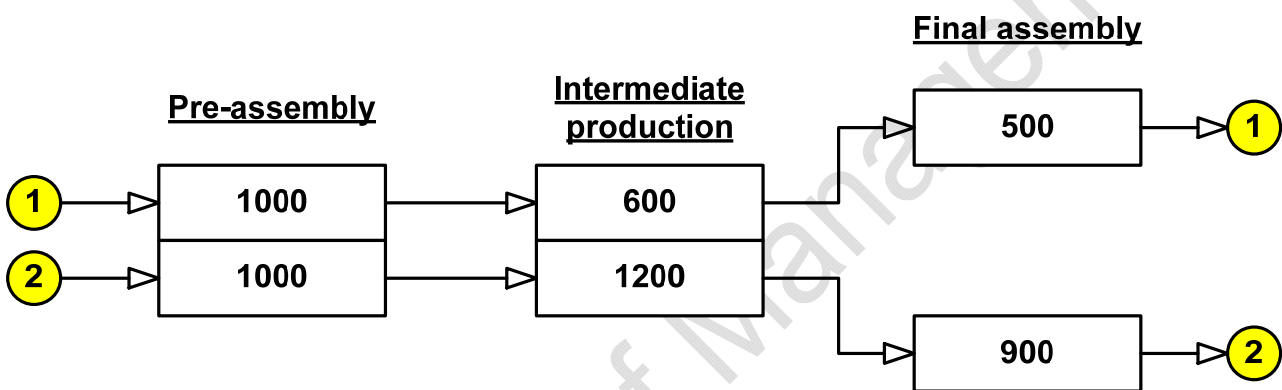
- **Criterion in the case of a single bottleneck = profit per unit of bottleneck consumption**

Products are accepted in descending order of profit per unit of bottleneck consumption. Production quantities correspond to maximum sales quantities. The final accepted product is produced up to the capacity limit.


◇ **Single-period master production scheduling (linear optimization model)**

Below, the production process of a vehicle is sketched. Vehicles of type 1 and 2 pass a three-stage production process consisting of pre-assembly, intermediate production, and final assembly. The latter is conducted on separate lines for each of the vehicle types. Production capacities are indicated in the figure below. In pre-assembly, per unit production times of the vehicle types are identical. In intermediate production, vehicle type 1 requires twice as much capacity per unit as type 2 so that total output varies between 600 vehicles, if only type 1 is produced and 1200 vehicles, if only type 2 is produced. Per unit profit is 6€ and 4€ for the two vehicle types, respectively.

Formulate the master production scheduling problem as a linear optimization model. First, define the decision variables.



**Decision variables**

**Model formulation**⇒ **Objective function**

(1) Please select the correct objective function formulation for the optimization model:

- a.  $\min 6x_1 + 4x_2$
- b.  $\max 6x_1 + 4x_2$
- c.  $\min 500x_1 + 900x_2$
- d.  $\max x_1 + x_2$

Subject to

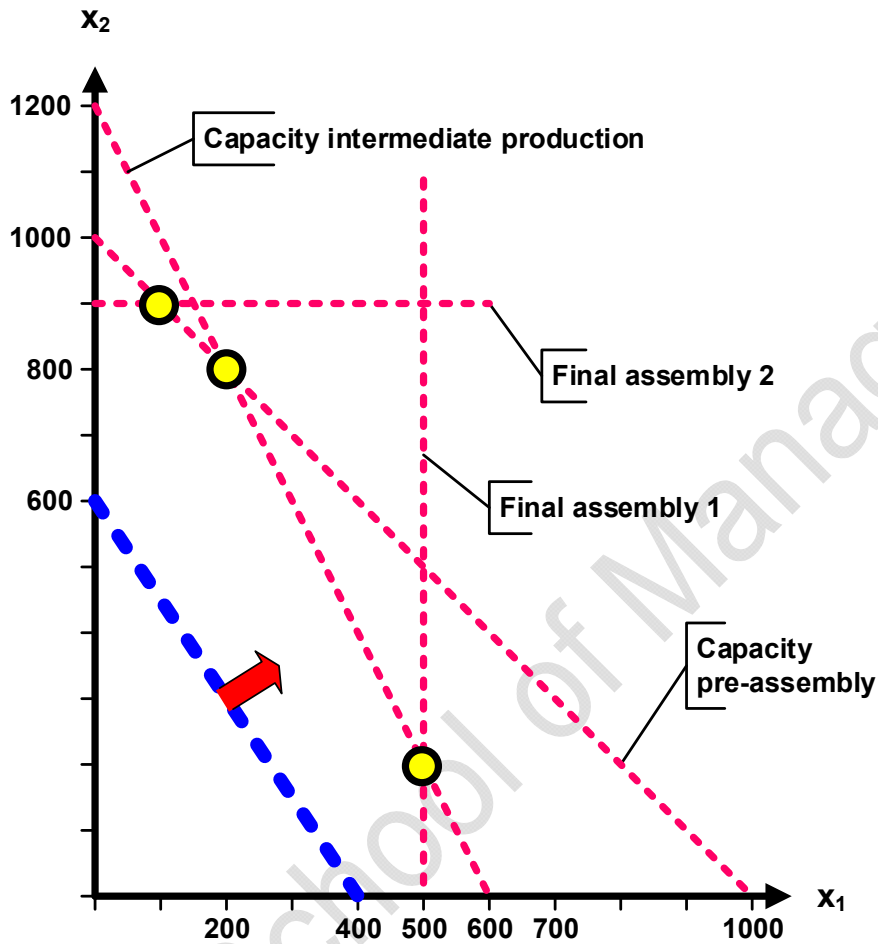
⇒ **Pre-assembly**⇒ **Intermediate production**⇒ **Final assembly**⇒ **Non-negativity**

**Graphical solution**

⇒ Profit line (Profit = 2400):

⇒ Optimal solution:

⇒ Maximum profit:



(2) Which is the optimal solution and objective function value (OFV)?

- a.  $x_{1 \text{ opt}} = 200$ ;  $x_{2 \text{ opt}} = 800$ ;  $OFV = 4.400$
- b.  $x_{1 \text{ opt}} = 100$ ;  $x_{2 \text{ opt}} = 900$ ;  $OFV = 4,200$
- c.  $x_{1 \text{ opt}} = 500$ ;  $x_{2 \text{ opt}} = 900$ ;  $OFV = 6.600$
- d.  $x_{1 \text{ opt}} = 150$ ;  $x_{2 \text{ opt}} = 900$ ;  $OFV = 4,500$



### Linear optimization model for multi-period master production scheduling

⇒ **Objective:** Minimization of inventory holding costs

#### **Assumptions**

- Storable products
- Constant per unit costs
- Given requirement quantities to be satisfied without shortages
- No extra capacity, e.g. overtime

#### **Model formulation**

- Indices, sets
  - $j \in J$  Products
  - $i \in I$  Resources
  - $t \in T$  Periods ( $t=1, \dots, T$ )
- Decision variables
  - $x_{jt}$  Production quantity of product  $j$  in period  $t$
  - $y_{jt}$  Inventory of product  $j$  at the end of period  $t$  ( $y_{j0} = \text{given}$ )
- Data
  - $a_{ij}$  Consumption of resource  $i$  per unit of product  $j$
  - $B_{it}$  Available capacity of resource  $i$  in period  $t$
  - $d_{jt}$  Forecasted demand of product  $j$  in period  $t$
  - $h_j$  Inventory holding costs per unit and period for product  $j$
- Optimization model  
Minimize the **objective function**

Subject to the following constraints:

#### **Capacity constraints**

#### **Inventory balances**

#### **Non-negativity**

◇ Example

Product	Forecasted demand in period				Capacity consumption per unit		Inventory holding costs per unit and period	Initial inventory
	1	2	3	4	Stage 1	Stage 2		
1	20	10	12	18	1.0	2.0	2	10
2	10	20	40	30	1.5	0.5	3	15
<b>Capacity</b>	-	-	-	-	<b>50</b>	<b>40</b>		

- **Decision variables**

$x_{jt}$  Production quantity of product  $j$  in period  $t$

$y_{jt}$  Inventory of product  $j$  at the end of period  $t$  ( $y_{j0} = \text{given}$ )

- **Model formulation**

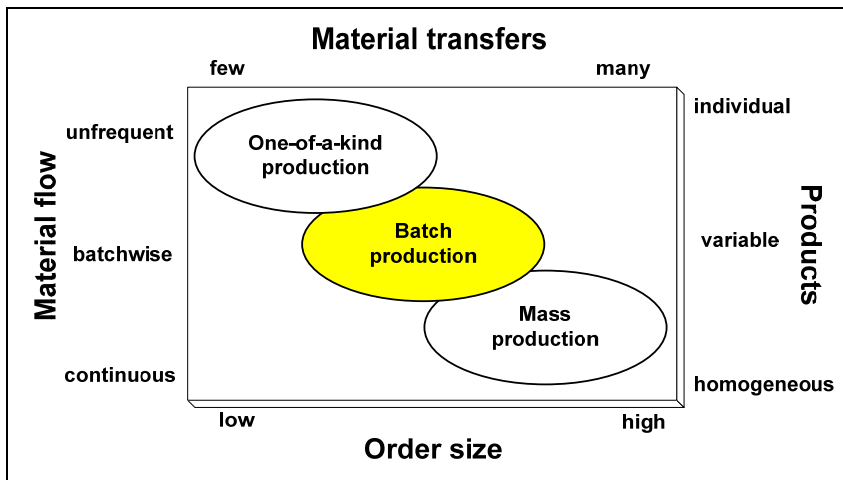
Objective function

Capacity

Inventory balances

Non-negativity

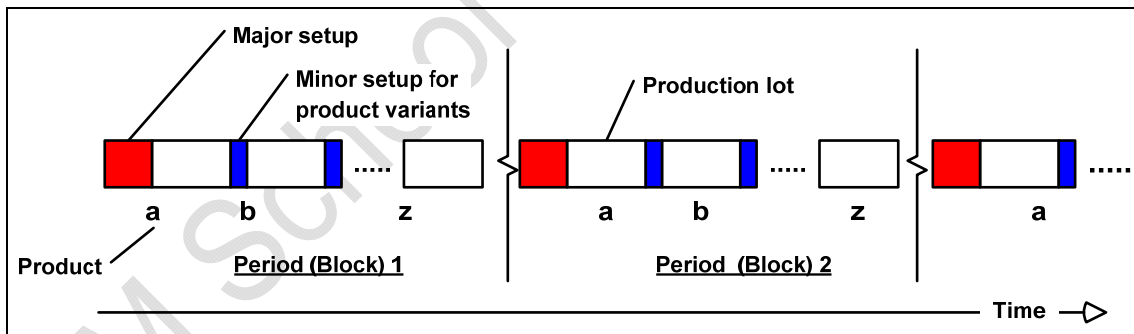
## 5.2 Batch production



- **Characteristics of batch production**

- ⇒ Different **variants of a basic product type** are manufactured using the same production equipment one after another.
- ⇒ Changeover times and costs, e.g. major setup for a product family and minor setup for product variants
- ⇒ Examples: food production, detergents, cosmetics
- ⇒ Make-to-stock or assemble-to-order

- **Block planning**



### Linear optimization model for multi-period master production scheduling

- ⇒ **Objective:** Minimization of setup and inventory holding costs

#### Assumptions regarding the model formulation

- ⇒ Storable products
- ⇒ Constant per unit costs
- ⇒ Given requirement quantities to be satisfied without shortages
- ⇒ **Single bottleneck resource**, e.g. make-and-pack production line

### Model formulation

- *Indices, sets*

$j \in J$  Products

$t \in T$  Periods ( $t=1, \dots, T$ )

- *Decision variables*

$x_{jt}$  Production quantity of product  $j$  in period  $t$

$y_{jt}$  Inventory of product  $j$  at the end of period  $t$  ( $y_{j0} = \text{given}$ )

$z_{jt} = 1$ , if product  $j$  is set up in period  $t$  (0, otherwise)

- *Data*

$a_j$  Capacity consumption per unit of product  $j$

$C_t$  Available production capacity in period  $t$

$d_{jt}$  Forecasted demand of product  $j$  in period  $t$

$h_j$  Inventory holding costs per unit and period for product  $j$

$s_j$  Setup time for product  $j$

$SC_j$  Setup costs for product  $j$

$BIG$  Large number (big M), e.g.  $BIG = \max_{j,t} \frac{C_t - s_j}{a_j}$

- **Optimization model**

Minimize the **objective function**

$$\sum_{j \in J} \sum_{t \in T} (SC_j \cdot z_{jt} + h_j \cdot y_{jt})$$

Subject to the following constraints:

#### Capacity constraints

$$\sum_{j \in J} (a_j \cdot x_{jt} + s_j \cdot z_{jt}) \leq C_t \quad \forall t \in T$$

#### Product setup

$$x_{jt} \leq BIG \cdot z_{jt} \quad \forall j \in J, t \in T$$

#### Inventory balances

$$y_{jt} = y_{j,t-1} + x_{jt} - d_{jt} \quad \forall j \in J, t \in T$$

#### Non-negativity and integrality conditions

$$x_{jt}, y_{jt} \geq 0 \quad \forall j \in J, t \in T$$

$$z_{jt} \in \{0, 1\} \quad \forall j \in J, t \in T$$



### Example: Production of three variants on the same equipment

Product	Forecasted demand in week			Capacity consumption per unit	Setup costs	Setup time	Inventory holding costs per unit and period	Initial inventory
	1	2	3					
1	5	8	6	1.0	20	3	2	2
2	3	2	4	1.2	30	1	4	3
3	4	1	2	1.5	40	1	3	1
<b>Capacity</b>	-	-	-	<b>15</b>				

- **Model formulation**


Objective function

Capacity constraints

Product setup

Inventory balances

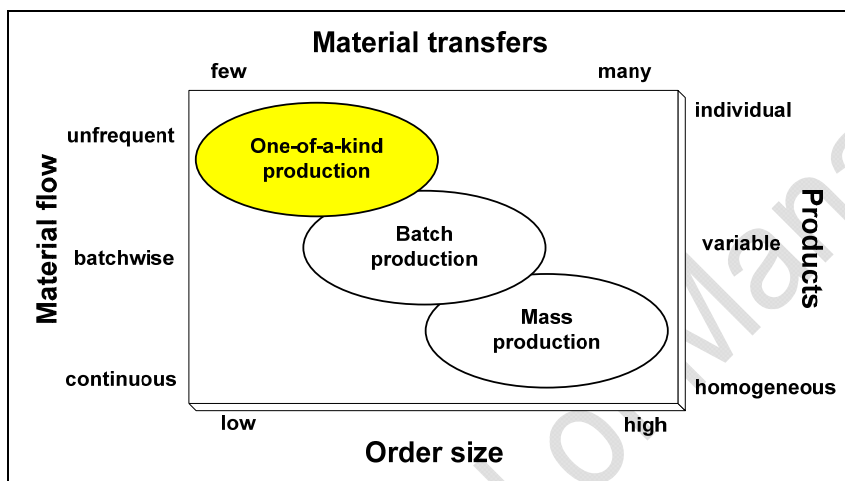
Non-negativity and integrality conditions



(5) What is a good value for the BIG-M-parameter?

- (1) 5,000,000
- (2) 8
- (3) 11.67
- (4) 12

### 5.3 One-of-a-kind production



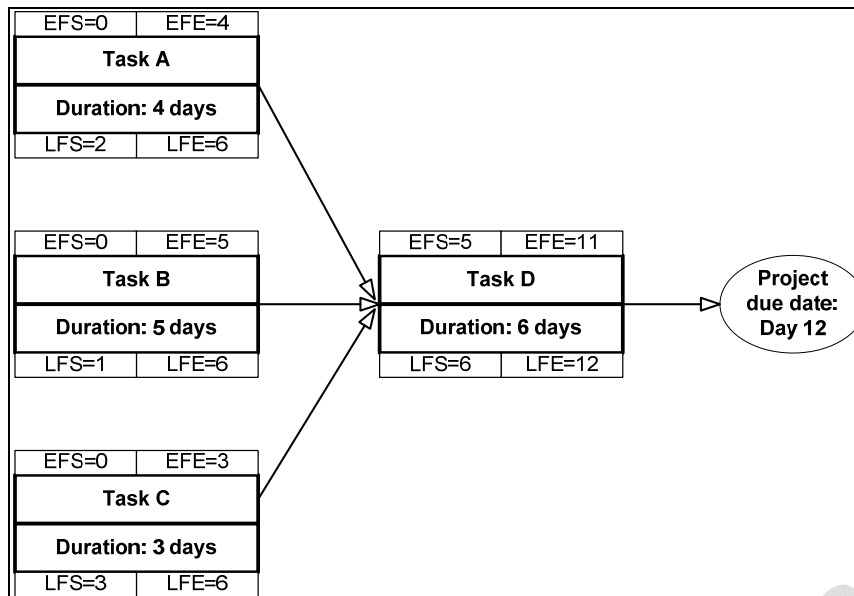
- **Characteristics**

- ⇒ Individual products that are produced in very small order sizes
- ⇒ Often customer-specific adjustment of product design
- ⇒ Only limited number of production orders at the same time in the system
- ⇒ Large work volume, numerous production tasks, complex project structure
- ⇒ Long throughput times and tight due dates

- **Additional planning tasks**

- ⇒ Preparation of bids
- ⇒ Planning of costs and resource requirements
- ⇒ Project structuring
- ⇒ Determination of feasible due dates
- ⇒ Concurrent monitoring of times, resource utilization and costs

- Time scheduling for a single work package



- Critical path method (CPM)

- ⇒ Notation

$EFS_j, EFE_j$  Earliest feasible start and end time of task  $j$

$LFS_j, LFE_j$  Latest feasible start and end time of task  $j$

$d_j$  Duration of task  $j$

- ⇒ Forward calculation

$$EFS_0 = 0$$

$$EFE_j = EFS_j + d_j$$

$$EFS_j = \max_{v \in V(j)} \{EFE_v\}, \text{ where } v \in V(j) = \text{predecessor of } j$$

- ⇒ Backward calculation

$$LFE_j = \begin{cases} \text{due date} \\ EFE_j, \text{ if no due date is given} \end{cases}, \text{ where } J = \text{final task}$$

$$LFS_j = LFE_j - d_j$$

$$LFE_j = \min_{n \in N(j)} \{LFS_n\}, \text{ where } n \in N(j) = \text{successor of } j$$

- ⇒ Total buffer time

$$TB_j = LFE_j - EFE_j = LFS_j - EFS_j$$

⇒ **Critical path:** Connects all tasks which together determine the shortest possible duration of the project, with  $TB_j = \begin{cases} 0, \text{ when no due date is given} \\ \text{due date} - EFE_j, \text{ when a due date is given} \end{cases}$

### ◇ Planning of a multi-stage production order

A company produces facility A which is composed of two units each of sub-assemblies B and C. Sub-assembly B consists of four units of part D, two units of part E and one unit of part F. Sub-assembly C consists of three units each of part F and G.

The various units of a part or sub-assembly are produced one after another, i.e. no overlapping takes place and the entire lot is supplied to the next stage after completion of the final unit. The following table indicates per unit production times. The due date for the completion of the project is after 40 time units.

Product	A	B	C	D	E	F	G
Per unit production time	4	2	7	1	3	2	3

- Display the structure of facility A graphically.
- How many units of sub-assembly B and C and how many units of parts D, E, F are needed?
- Develop the graphical time representation of the project using a network. Determine the earliest feasible end time of the project and the latest feasible end time of tasks based on the deadline of 40 time units.

### Solution

- Structure of the facility

- Product requirements

Product	A	B	C	D	E	F	G
Requirement							

c) Network

Product	A	B	C	D	E	F	G
Total buffer time							

(6) Which are the earliest end dates for the production of the parts and the end product? The values are sorted in alphabetical order of the parts and products from A to G.

- a. 36,20,32,8,12,16,18
- b. 36,16,32,8,12,16,18
- c. 24,20,32,8,12,16,18
- d. 36,20,30,8,12,16,18

(7) What is the latest start date for the production of the parts and the end product to fulfil the deadline of 40 days?

- a. 36,32,22,24,20,16,4
- b. 36,32,22,24,20,6,4
- c. It is not possible to finish the product in 40 days
- d. 36,43,22,24,20,6,0