

Investment & Financial Management Formulary

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INTEREST CALCULATION

C_0 Initial Capital (Present Value)
 C_N Final Capital (Future Value)
 N, n Number of Total Interest Payments / Maturity in Years
 m Number of Interest Payments per Year
 r, r_{eff} Interest Rate (nominal / effective)
Simple Interest: $C_N = C_0 \cdot (1 + N \cdot r)$
Compound Interest: $C_N = C_0 \cdot (1 + r)^N$
Intra-Year Interest: $N = m \cdot n$

$$r_{eff} = \left(1 + \frac{r_{\text{yearly}}}{m}\right)^m - 1$$

Continuous Interest: $C_N = C_0 \cdot e^{r \cdot n}$

$$r_{eff} = e^r - 1$$

ANNUITY CALCULATION

C, C_{due} Annuity-immediate / Annuity-due
 N Maturity
 m Number of Interest Payments per Year
 r, q Interest Rate / Compounding Factor ($q = 1 + r$)
 w Annual Growth Rate of annuity, given geo. growing annuity
 g Growth Factor ($g = 1 + w$)
 d Increase in annuity, given arith. growing annuity

Constant Annuity

annuity-immediate **annuity-due**
Final Value $FV = C \cdot \frac{q^N - 1}{q - 1}$ $FV_{due} = C_{due} \cdot q \cdot \frac{q^N - 1}{q - 1}$
Present Value $PV = C \cdot \frac{1}{q^N} \cdot \frac{q^N - 1}{q - 1}$ $PV_{due} = C_{due} \cdot \frac{1}{q^{N-1}} \cdot \frac{q^N - 1}{q - 1}$

Arithmetically Growing Annuity

annuity-immediate
Final Value $FV = \left(C + \frac{d}{q-1}\right) \cdot \frac{q^N - 1}{q-1} - \frac{N \cdot d}{q-1}$
Present Value $PV = \left(C + \frac{d}{q-1}\right) \cdot \frac{q^N - 1}{q^N \cdot (q-1)} - \frac{N \cdot d}{q^N \cdot (q-1)}$
annuity-due
Final Value $FV_{due} = \left(C_{due} + \frac{d}{q-1}\right) \cdot \frac{q \cdot (q^N - 1)}{q-1} - \frac{N \cdot d \cdot q}{q-1}$
Present Value $PV_{due} = \left(C_{due} + \frac{d}{q-1}\right) \cdot \frac{q^N - 1}{q^{N-1} \cdot (q-1)} - \frac{N \cdot d}{q^{N-1} \cdot (q-1)}$

Geo. Growing Annuity ($g = 1 + w$ and $q = 1 + r$)

annuity-immediate
Final Value $FV = \begin{cases} C \cdot N \cdot q^{N-1} & g = q \\ C \cdot \frac{q^N - q^N}{g - q} & g \neq q \end{cases}$
Present Value $PV = \begin{cases} \frac{C \cdot N}{q} & g = q \\ C \cdot \frac{(q/q)^N - 1}{g - q} & g \neq q \end{cases}$
annuity-due
Final Value $FV_{due} = \begin{cases} C_{due} \cdot N \cdot q^N & g = q \\ C_{due} \cdot q \cdot \frac{q^N - q^N}{g - q} & g \neq q \end{cases}$
Present Value $PV_{due} = \begin{cases} C_{due} \cdot N & g = q \\ C_{due} \cdot q \cdot \frac{(q/q)^N - 1}{g - q} & g \neq q \end{cases}$

Perpetual Annuity (Present Value)

annuity-immediate **annuity-due**
constant $PV = \frac{C}{r}$ $PV_{due} = q \cdot \frac{C_{due}}{r}$
growing ($w < r$) $PV = \frac{C}{q-g} = \frac{R}{r-w}$ $PV_{due} = q \cdot \frac{C_{due}}{q-g} = q \cdot \frac{C_{due}}{r-w}$

Fictitious periodic annuity / Replacement Annuity

annuity-immediate **annuity-due**
Fict. p. annuity $C' = C \cdot \left[m + \frac{r \cdot (m-1)}{2}\right]$ $C' = C_{due} \cdot \left[m + \frac{r \cdot (m+1)}{2}\right]$

REPAYMENT CALCULATION

A_k Annuity payment in period k ($A_k = T_k + I_k$)
 N, k Maturity / Foregone Periods
 D_0 Initial Debt ($D_0 = \sum_{k=1}^N T_k$)
 D_k Debt in Period k ($D_k = D_{k-1} - T_k$)
 T_k, I_k Redemption / Interest Payment in Period k ($I_k = r \cdot D_{k-1}$)
 q Compounding factor ($q = 1 + r$)
Installment Repayment **Annuity Repayment**
 $A_k = D_0 \cdot \left(r \cdot \left(1 - \frac{k-1}{N}\right) + \frac{1}{N}\right)$ $A = D_0 \cdot \frac{q^N (q-1)}{q^N - 1}$
 $D_k = D_0 \cdot \left(1 - \frac{k}{N}\right)$ $D_k = D_0 \cdot \frac{q^N - q^k}{q^N - 1}$

BOND VALUATION

B_0, B_0^{ZB} Present Value of a Coupon Bond / Zero Bond
 B_N Repayment (Face Value)
 C Coupon
 N Maturity
 r Interest Rate
 D Duration ($D = \frac{1}{B_0} \cdot \left[\sum_{k=1}^N \frac{k \cdot C_k}{(1+r)^k} + \frac{N \cdot B_N}{(1+r)^N}\right]$)
 D_{mod} Modified Duration ($D_{mod} = \frac{1}{1+r} \cdot D$)
 $I_{t_1, t_2}, r_{t_1, t_2}$ Spot Rate / Forward Rate from t_1 to t_2
Coupon Bond: $B_0 = \left[C \frac{(1+r)^N - 1}{r} + B_N\right] \frac{1}{(1+r)^N}$
Zerobond: $B_0^{ZB} = \frac{B_N}{(1+r)^N}$
Approximation of the Change in PV: $\frac{\Delta B_0}{B_0} \approx -D_{mod} \cdot \Delta r$
Term Structure $(1 + I_{t,T})^{T-t} = (1 + I_{t,S})^{S-t} \cdot (1 + r_{S,T})^{T-S}$

STOCK VALUATION

P_t, D_t Price of the Stock / Dividend at time t
 $(d)EPS$ (diluted) Earnings per Share
 r_e Equity Cost of Capital
 P/E Price/Earnings-Ratio ($P/E = \frac{P_0}{EPS}$)
 P/B Price-to-Book Ratio ($P/B = \frac{P_0}{V_E}$)
 V_E Book Value of Equity
 w Growth Rate ($w = (1 - p) \cdot ROE$)
 p Payout Ratio
 ROE Return on Equity

Valuation

Total Return (1Y) $r_e = \frac{D_1 + P_1}{P_0} - 1 = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0}$
Dividend Discount Model $P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_e)^t}$
Constant Growth $P_0 = \frac{D_1}{r_e - w}$
 $P_0 = \sum_{t=1}^N \frac{D_1 \cdot (1+w_a)^{t-1}}{(1+r_e)^t}$
Changing Growth $+ \frac{D_1 \cdot (1+w_a)^{N-1} \cdot (1+w_b)}{r_e - w_b} \cdot \frac{1}{(1+r_e)^N}$

OPTION VALUATION AND BINOMIAL MODEL

C Value of a Call Option ($C = \frac{pC_u + (1-p)C_d}{1+r_{RF}}$)
 P Value of a Put Option ($P = \frac{pP_u + (1-p)P_d}{1+r_{RF}}$)
 r_{RF} Risk-free Interest Rate
 $u - 1$ Increase in Stock Price
 $1 - d$ Decrease in Stock Price
 p Probability of Increase (risk-neutral) ($p = \frac{(1+r_{RF}) - d}{u - d}$)
 $1 - p$ Probability of Decrease (risk-neutral)

FINANCIAL ANALYSIS

Definition of the Free Cash Flow to the Firm

Earnings before Interest and Taxes	EBIT
- Adjusted Tax Expense	$\tau_C \cdot \text{EBIT}$
= Net Operating Profit After Taxes	NOPAT
+ Depreciation and amortization	Depr
- Capital Expenditure	CAPEX
\pm Change in Net Working Capital	ΔNWC
= Cash Flow from Operations	OCF
+ Cash Flow from Investments	CFI
= Free Cash Flow to the Firm	FCF

Definition of the Free Cash Flow to Equity

Net Income	NI
+ Depreciation and amortization	Depr
+ Other Non-Cash-Expenses	NCExp
- Other Non-Cash-Earnings	NCEarn
\pm Change in Net Working Capital	ΔNWC
= Cash Flow from Operations	OCF
+ Cash Flow from Investments	CFI
+ Cash Flow from Financing	CFF
= Free Cash Flow to Equity	FCFE

Important Financial Ratios

Net Debt (NFO)	Debt - Financial Assets
Invested Capital (IC)	Book Value of Equity + NFO
Debt-to-Capital Ratio	$\frac{D}{E+D}$
Debt-Equity Ratio (L)	$\frac{D}{E}$
Net Financial Leverage (NFL)	$\frac{\text{NFO}}{E}$
Debt-to-EV Ratio	$\frac{\text{NFO}}{\text{MV Equity} + \text{NFO}}$
Interest Coverage Ratio	$\frac{\text{EBIT (EBITDA)}}{\text{Interest Expense}}$
Current Ratio	$\frac{\text{Current Assets}}{\text{Current liabilities}}$
Quick Ratio	$\frac{\text{Cash} + \text{short-term investments} + \text{A/R}}{\text{Current liabilities}}$
Return on Equity (ROE)	$\frac{\text{NI}}{E}$
Return on Invested Capital (ROIC)	$\frac{\text{EBIT}(1-\text{Tax rate})}{\text{IC}}$
Net Financial Expense (NFE)	$\frac{\text{FE}}{\text{NFO}}$
EBIT Margin	$\frac{\text{EBIT}}{\text{Sales}}$
Net Profit Margin	$\frac{\text{NI}}{\text{Sales}}$
Earnings per Share (EPS)	$\frac{\text{NI}}{\# \text{ of shares outstanding}}$
Diluted EPS (dEPS)	$\frac{\text{NI}}{\# \text{ of shares outstanding} \cdot \text{dilution factor}}$
Price-Earnings-Ratio (P/E)	$\frac{\text{Stock price}}{\text{EPS}} = \frac{\text{market capitalization}}{\text{NI}}$
EBITDA Multiple	$\frac{\text{EV}}{\text{EBITDA}}$
Market to Book Ratio	$\frac{\text{Stock price} \cdot \# \text{ of shares outstanding}}{\text{book value of equity}}$

Diluted Earnings

<i>EPS</i>	earnings per share
<i>a</i>	number of outstanding shares
<i>n</i>	number of new shares to be issued
<i>X, S</i>	issue (exercise) / current stock price
<i>NV</i>	nominal value of convertible bond
<i>c</i>	coupon rate
<i>NI</i>	net income
τ_C	Tax rate

Calculation:

$$EPS(\text{dilluted}) = \frac{EPS}{DF1 \cdot DF2}$$

$$DF1 = \max \left[1; 1 + \frac{n}{a} \left(1 - \frac{X}{S} \right) \right]$$

$$DF2 = \max \left[1; \frac{1 + \frac{n}{a}}{1 + \frac{NV(1-\tau_c)}{NI}} \right]$$

Breakdown-Analysis

<i>ROE</i>	operating Return on Equity
<i>ROIC^{aT}</i>	Return on Invested Capital after Tax
<i>NFL</i>	Net Financial Leverage
<i>NFE</i>	Net Financial Expense
<i>NFO</i>	Net Debt (=Net Financial Obligation)
τ_C	Tax rate

$$ROE = ROIC^{aT} + NFL$$

Book-Leverage Equation

$$\cdot \left[ROIC^{aT} - \frac{NFE \cdot (1 - \tau_C)}{NFO} \right]$$

INVESTMENT ANALYSIS

<i>NPV</i>	Net Present Value ($NPV = \sum_{t=0}^T \frac{CF_t}{(1+r)^t}$)
<i>t, T</i>	time index / Number of periods
<i>CF_t</i>	Net payment in period <i>t</i>
<i>r</i>	Discount rate
<i>IRR</i>	Internal rate of return ($\sum_{t=0}^T \frac{CF_t}{(1+IRR)^t} \stackrel{!}{=} 0$)
τ_C	Tax rate

Analysis:

Profitability Index	$PI = \frac{NPV}{\text{Resources consumed}}$
Economic Value Added (EVA)	$EVA_t = EBIT_t \cdot (1 - \tau_C) - r \cdot IC_{t-1}$

CAPITAL INCREASE

<i>P_{cum}</i>	Price of the old share (Price cum)
<i>IP</i>	Subscription price of the new shares
<i>BV</i>	Subscription ratio (old shares / new shares)
<i>Div_a</i>	Dividend old share
<i>Div_n</i>	Dividend new share
<i>SR</i>	Value of the subscription right
<i>P_{ex}</i>	Share price immediately after the capital increase
<i>t</i>	Years until the next dividend payment
<i>q</i>	$1 + r_E = 1 + \text{equity cost of capital}$

Capital increase against cash contribution

Calculated price ex	$P_{ex} = \frac{BV \cdot P_{cum} + IP}{BV+1}$
Value of the subscription right	$SR = \frac{P_{cum} - IP}{BV+1}$
Value of the subscription right with dividend disadvantage for the new shares	$SR = \frac{P_{cum} - IP - (Div_a - Div_n) \cdot q^{-t}}{BV+1}$

Capital increase from company funds

$$\text{Calculated price ex } P_{ex} = \frac{BV \cdot P_{cum}}{BV+1} = \frac{P_{cum}}{1 + \frac{1}{BV}}$$

COST OF CAPITAL AND CAPM

<i>E, r_E</i>	Value of equity / equity cost of capital
<i>D, r_D</i>	Value of debt / debt cost of capital
<i>r_U, r_{WACC}</i>	Cost of capital of the unlevered / levered firm
<i>r_{RF}</i>	Risk free rate
\bar{r}_i, \bar{r}_{Mkt}	Expected return of the asset <i>i</i> / the market portfolio
β_i	Beta factor of the asset <i>i</i>
β_E	Beta factor of equity (equity beta)
β_D	Beta factor of debt (debt beta)
β_U	Beta factor of assets (unlevered beta)
<i>V^U, V^L</i>	Value of the unlevered / levered firm
<i>CAPM</i>	$\bar{r} = r_{RF} + \beta_i(\bar{r}_{Mkt} - r_{RF})$

Cost of capital without taxes

$$\text{Cost of capital of the unlevered firm } r_U = \frac{E}{E+D} r_E + \frac{D}{D+E} r_D$$

$$\text{Equity cost of capital of the levered firm } r_E = r_U + \frac{D}{E} (r_U - r_D)$$

$$\text{Unlevered beta } \beta_U = \frac{E}{E+D} \beta_E + \frac{D}{E+D} \beta_D$$

$$\text{Equity beta } \beta_E = \beta_U + \frac{D}{E} (\beta_U - \beta_D)$$

Cost of capital with taxes

$$\text{Weighted Average Cost of Capital } r_{WACC} = \frac{E}{E+D} r_E + \frac{D}{D+E} r_D (1 - \tau_C)$$

$$\text{WACC with target leverage ratio } r_{WACC} = r_U - \frac{D}{E+D} \tau_C r_D$$

$$\text{Unlevered beta (Hamada equation) } \beta_U = \frac{\beta_E}{1 + \frac{D}{E} (1 - \tau_C)}$$

$$\text{Levered firm value } V^L = V^U + PV(\text{Interest tax shield})$$

CORPORATE VALUATION

<i>E, r_E</i>	Value of equity / equity cost of capital
<i>D, r_D</i>	Value of debt / cost of capital
<i>V</i>	Firm value
<i>r_U, r_{WACC}</i>	Cost of capital of the unlevered / levered firm
τ_C	Marginal tax rate
<i>FCFE</i>	Expected Free Cash Flow to Equity
<i>FCF</i>	Expected Free Cash Flow to Firm
Valuation	
Equity method	$E = \sum_{t=1}^{\infty} FCFE_t (1 + r_E)^{-t}$
Entity method	$V = \sum_{t=1}^{\infty} FCF_t (1 + r_{WACC})^{-t}$
	$V = \sum_{t=1}^{\infty} FCF_t (1 + r_U)^{-t} + \tau_C \sum_{t=1}^{\infty} D \cdot r_D (1 + r_D)^{-t}$
APV method	$= \sum_{t=1}^{\infty} FCF_t (1 + r_U)^{-t} + \tau_C D$